

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI - 600034

## M.Sc. DEGREE EXAMINATION - STATISTICS

FOURTH SEMESTER - NOVEMBER 2013

## ST 4811 - ADVANCED OPERATIONS RESEARCH

Date : 06/11/2013
Dept. No. $\square$ Max. : 100 Marks
Time : 1:00-4:00

## SECTION A

Answer ALL questions. Each carries two marks. $(2 \times 10=20)$

1. Define a General Linear Programming Problem.
2. Define feasible solution in a Linear Programming Problem.
3. Define a general primal dual pair.
4. What are the applications of goal programming?
5. State the principal of optimality in dynamic programming.
6. Distinguish between Pure and Mixed Integer Programming Problems?
7. What is a Non Linear Programming Problem?
8. Define a quadratic programming problem.
9. What is inventory control?
10. What is queue discipline?

## SECTION B

Answer any FIVE questions. Each carries eight marks.
11. Solve the following LPP; Minimize $Z=-3 X_{1}-2 X_{2}$, subject to the constraints,

$$
\mathrm{X}_{1}-\mathrm{X}_{2} \leq 1 ; 3 \mathrm{X}_{1}-2 \mathrm{X}_{2} \leq 6 ; \text { and } \mathrm{X}_{1}, \mathrm{X}_{2} \geq 0 .
$$

12. Describe the Gomory's constraint method, and derive Gomory's constraint for solving a Pure Integer Programming Problem.
13. Solve the following non linear programming Problem;
$\operatorname{Max} \mathrm{Z}=\mathrm{X}_{1}{ }^{2}+\mathrm{X}_{2}{ }^{2}+\mathrm{X}_{3}{ }^{2}$ subject to the constraints, $\mathrm{X}_{1}+\mathrm{X}_{2}+\mathrm{X}_{3}=1$; and $\mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{X}_{3} \geq 0$.
14. Test for extreme values of $f\left(x_{1}, x_{2}, x_{3}\right)=x_{1}^{2}+x_{2}^{2}+x_{3}^{2}$, subject to the constraints,
$x_{1}+x_{2}+3 x_{3}=2$ and $5 x_{1}+2 x_{2}+x_{3}=5$.
15. Using Dynamic Programming Problem, maximize $\mathrm{z}=\left\{\mathrm{y}_{1} \cdot \mathrm{y}_{2} \ldots . . \mathrm{y}_{\mathrm{n}}\right\}$ subject to the constraints, $\mathrm{y}_{1}+\mathrm{y}_{2}$ $+. \ldots . .+y_{n}=c$, and $y_{j} \geq 0$.
16. A corporation is entertaining proposals from its 3 plants for possible expansion of its facilities. The corporation's budget is $£ 5$ millions for allocation to all 3 plants. Each plant is requested to submit its proposals giving total cost C and total revenue R for each proposal. The following table summarizes the cost and revenue in millions of pounds. The zero cost proposals are introduced to allow for the probability of not allocating funds to individual plants. The goal of the corporation is to maximize the total revenue resulting from the allocation of $£ 5$ millions to the three plants.

|  | Plant 1 |  |  | Plant 2 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Plant |  |  |  |  |  |  |
| Proposal | $\mathbf{C}_{\mathbf{1}}$ | $\mathbf{R}_{\mathbf{1}}$ | $\mathbf{C}_{\mathbf{2}}$ | $\mathbf{R}_{\mathbf{2}}$ | $\mathbf{C}_{\mathbf{3}}$ | $\mathbf{R}_{\mathbf{3}}$ |
| $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| $\mathbf{2}$ | $\mathbf{1}$ | $\mathbf{5}$ | $\mathbf{2}$ | $\mathbf{8}$ | $\mathbf{1}$ | $\mathbf{3}$ |
| $\mathbf{3}$ | $\mathbf{2}$ | $\mathbf{6}$ | $\mathbf{3}$ | $\mathbf{9}$ | - | - |
| $\mathbf{4}$ | - | - | $\mathbf{4}$ | $\mathbf{1 2}$ | - | - |

Use Dynamic Programming Problem to obtain the optimal policy for the above problem.
17. An electronic device consists of 4 components, each of which must function for the system to function. The system reliability can be improved by installing parallel units in one or more of the components. The reliability R of a component with 1,2 or 3 parallel units and the corresponding cost C (in '000s) are given in the following table. The maximum amount available for this device is Rs.
$1,00,000$. Determine the number of Parallel units in each component.

|  | $\mathrm{j}=1$ |  | $\mathrm{j}=2$ |  | $\mathrm{j}=3$ |  | $\mathrm{j}=4$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of unit | $\mathrm{R}_{1}$ | $\mathrm{C}_{1}$ | $\mathrm{R}_{1}$ | $\mathrm{C}_{1}$ | $\mathrm{R}_{1}$ | $\mathrm{C}_{1}$ | $\mathrm{R}_{1}$ | $\mathrm{C}_{1}$ |
| 1 | . 7 | 10 | . 5 | 20 | . 7 | 10 | . 6 | 20 |
| 2 | . 8 | 20 | . 7 | 40 | . 9 | 30 | . 7 | 30 |
| 3 | . 9 | 30 | . 8 | 50 | . 95 | 40 | . 9 | 40 |

18. Explain the important characteristics of a queuing system.

## SECTION C

## Answer any TWO questions. Each carries twenty marks.

19. Find an optimum integer solution to the following LPP: Mazimize $Z=X_{1}+4 X_{2}$, subject to the constraints, $2 \mathrm{X}_{1}+4 \mathrm{X}_{2} \leq 7,5 \mathrm{X}_{1}+3 \mathrm{X}_{2} \leq 15$ and $\mathrm{X}_{1}, \mathrm{X}_{2}$ are non-negative integers.
20. Solve the following Non Linear Programming Problem: $\operatorname{Max} Z=7 X_{1}{ }^{2}+6 X_{1}+5 X_{2}{ }^{2}$ subject to the constraints, $X_{1}+2 X_{2} \leq 10 ; X_{1}-3 X_{2} \leq 9 ;$ and $X_{1}, X_{2} \geq 0$,
21. Solve the following Quadratic programming Problem, by Wolfe's algorithm.

Max $Z=4 X_{1}+6 X_{2}-2 X_{1} X_{2}-2 X_{1}^{2}-2 X_{2}^{2}$ subject to the constraints,
$\mathrm{X}_{1}+2 \mathrm{X}_{2} \leq 2 ; \mathrm{X}_{1}, \mathrm{X}_{2} \geq 0$.
22. (i) For a (M/M/1) : ( $\infty /$ FIFO) queuing model in the steady-state case, derive the steady state difference equations and obtain expressions for the mean and variance of queue length in terms of the parameters $\lambda$ and $\mu$.
(ii) Explain the classical static Economic Order Quantity model and derive the expressions for Total Cost per Unit, order quantity, ordering cycle and effective lead time. $\quad(10+10)$

